SPRING 2025 MATH 590: QUIZ 5

Name:

1. For the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, find the eigenvalues of A and a basis for each associated eigenspace. (10 points)

Solution. We have $p_A(x) = \begin{vmatrix} x-1 & 0 & -1 \\ 0 & x-1 & 0 \\ -1 & 0 & x-1 \end{vmatrix} = (x-1)\cdot((x-1)^2-1) = x(x-1)(x-2)$, so the eigenvalues are: 0, 1, 2.

E_0 is the null space of $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \xrightarrow{\text{EROs}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. The rank of this matrix is two, so its null space has dimension one. Then $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ is a basis for the null space and hence a basis for E_0.

E_1 is the null space of $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \xrightarrow{\text{EROs}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, so $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ is a basis for E_1.

E_2 is the null space of $\begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \xrightarrow{\text{EROs}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, so $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ is a basis for E_2.